# Magnetothermopower and magnon-assisted transport in ferromagnetic tunnel junctions

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We present a model of the thermopower in a mesoscopic tunnel junction between two ferromagnetic metals based upon magnon-assisted tunneling processes. In our model, the thermopower is generated in the course of thermal equilibration between two baths of magnons, mediated by electrons. We predict a particularly large thermopower effect in the case of a junction between two half-metallic ferromagnets with antiparallel polarizations, $S_{AP} \sim -(k_B/e)$, in contrast to $S_P \approx 0$ for a parallel configuration.

Spin valve systems and magnetic multilayers displaying giant magnetoresistance effects also exhibit substantial magnetothermopower[1–6] with a strong temperature dependence. In metals, the thermopower $S$ is related to the conductivity of electrons taken at a certain energy, $\sigma(\epsilon)$, by the Mott formula,[7] $S = -(\pi^2 k_B^2 T/3e)\left(\partial \ln \sigma(\epsilon)/\partial\epsilon\right)_{\epsilon_F}$, so that it typically contains a small parameter such as $k_BT/\epsilon_F$. Theories of transport in magnetic multilayers with highly transparent interfaces based upon the use of the Mott formula have explained the difference between thermopower in the parallel (P) and anti-parallel (AP) configuration of ferromagnetic layers as due to either the difference in the energy dependence of the density of states for majority and minority spin bands in ferromagnetic layers,[8,9] or a different efficiency of electron-magnon scattering for carriers in opposite spin states.[3] In particular, the electron-magnon interaction in a ferromagnetic layer was incorporated to explain the observation[3] of a strong temperature dependence of $S(T)$ and gave, theoretically, a much larger thermopower in the parallel configuration of multilayers with highly transparent interfaces than in the antiparallel one, $S_P \gg S_{AP}$.

In this paper we investigate a model of the electron-magnon interaction assisted thermopower in a mesoscopic size ferromagnet/insulator/ferromagnet tunnel junction, which yields a different prediction. In the model we study below, the bottle-neck of both charge and heat transport lies in a small-area tunnel contact between ferromagnetic metals held at different temperatures, $T \pm \Delta T/2$. The thermopower is generated in the course of thermal equilibration between two baths of magnons, mediated by electrons. We find that the magnetothermopower effect is most pronounced in the case of half-metallic ferromagnets, where the exchange spin splitting $\Delta$ between the majority and minority conduction bands is greater than the Fermi energy $\epsilon_F$ measured from the bottom of the majority band, and the Fermi density of states in the minority band is zero. In a highly resistive antiparallel configuration of such a junction, where the emission/absorption of a magnon would lift the spin-blockade of electronic transfer between ferromagnetic metals, we predict a large thermopower effect, whereas in the lower-resistance parallel configuration thermopower appears to be relatively weak:

$$S_{AP} \approx -0.64\frac{k_B}{e}; \quad \frac{S_P}{S_{AP}} \sim \frac{k_BT}{\epsilon_F}. \tag{1}$$

We also found that for a junction between two conventional ferromagnetic metals, the ability of electronic transfer assisted by magnon emission/absorption to create thermopower depends on the difference between the size of majority/minority band Fermi surfaces and it is reduced by a temperature dependent factor $g(T) \sim (k_BT/\omega_D)^{3/2}$. The latter reflects the fractional change in the net magnetization of the reservoirs due to thermal magnons (Bloch's $T^{3/2}$ law).

Below, we describe the calculation of the thermopower for the case of a tunnel contact between two half-metallic ferromagnets and, then, we present its generalization to conventional ferromagnetic metals. We obtain an expression for the current $I(V, \Delta T)$ between bulk ferromagnetic reservoirs, as a function of bias voltage, $V$, and of the temperature drop, $\Delta T$, and, then, determine the thermopower coefficient $S = -V/\Delta T$ by satisfying the relation $I(V, \Delta T) = 0$. The expression for the current was derived using the balance equation, which takes into account competing elastic and inelastic electron transfer processes across the tunnel junction.

Let us consider, first, the AP configuration of ferromagnetic electrodes, with spin-$\uparrow$ majority electrons on the left hand side of the junction and spin-$\downarrow$ on the right. For such an alignment, elastic tunneling of carriers between electrodes is blocked by the absence of available states for a spin-polarized electron on the other side of an insulating barrier, whereas electron transfer may happen via tunneling processes assisted by a simultaneous emission/absorption of a magnon.[10] Since tails of wavefunctions of majority-spin ($\uparrow$) electrons close to the Fermi level on the left hand side penetrate into the forbidden region on the right, an electron on one side of the junction acquires a weak coupling with core magnetic moments (and, therefore, magnons) on the other side. A characteristic event can be viewed as a two-step quantum process. First, an electron tunnels into a virtual intermediate high-energy state in the minority band. Then,

it incorporates itself into the majority band by flipping spin in a magnon-emission process. Following the tunneling Hamiltonian approach,[11] the amplitude for a spin-$\uparrow$ electron with wave number $\mathbf{k}$ on the left to finish in a state $(\downarrow, \mathbf{k}')$ on the right after emitting a spin-wave with wavenumber $\mathbf{q}$ can be estimated using second order perturbation theory with respect to the electron - magnon interaction and the tunneling matrix element $t_{\mathbf{k},\mathbf{k}'+\mathbf{q}}$:

$$A_{\mathbf{k},\mathbf{k}'+\mathbf{q}} = \frac{t_{\mathbf{k},\mathbf{k}'+\mathbf{q}}\,\Delta}{\sqrt{2\xi\mathcal{N}}\,(\Delta + \epsilon_{\mathbf{k}'+\mathbf{q}} - \epsilon_{\mathbf{k}})} \approx \frac{t_{\mathbf{k},\mathbf{k}'+\mathbf{q}}}{\sqrt{2\xi\mathcal{N}}}, \qquad (2)$$

For $k_BT, eV \ll \Delta$, when both initial and final electron states should be taken close to the Fermi level, only long wavelength magnons can be emitted, so that the energy deficit in the virtual states can be approximated as $\Delta + \epsilon_{\mathbf{k}'+\mathbf{q}} - \epsilon_{\mathbf{k}} \approx \Delta$. As noticed in Refs. 10,12, this cancels out the large exchange parameter since the same electron-core spin exchange constant appears both in the splitting between minority and majority bands and in the electron-magnon coupling. [The number of localized moments in a ferromagnet $\mathcal{N}$ appears in Eq. (2) as we normalize both electron and magnon plane waves to the system volume, and $\xi$ is spin per unit magnetic cell.]

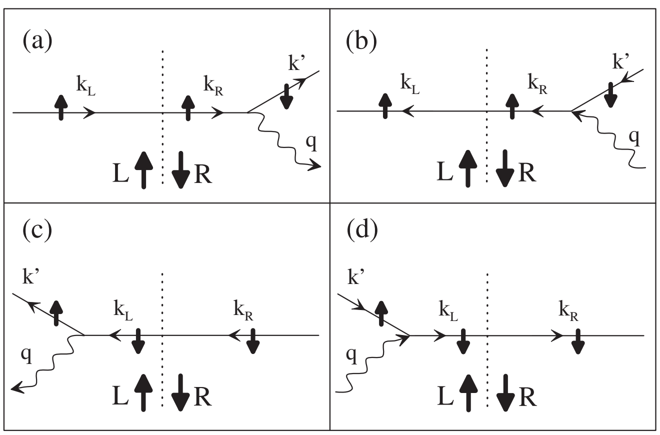


FIG. 1. Schematic of magnon-assisted tunneling across a junction with half-metallic electrodes in the anti-parallel configuration. Four processes which, to lowest order in the electron-magnon interaction, contribute to magnon-assisted tunneling. (a) and (c) involve magnon emission on the right and left hand sides, respectively, whereas (b) and (d) involve magnon absorption on the right and left.

To complete the balance equation describing electron transfer between half-metallic electrodes, one has to take into account four magnon-assisted tunneling processes depicted in Figure 1. Below, we describe them in detail assuming that the tunnel barrier is flat, so that the parallel component of the electron momentum conserves upon tunneling. Two of these processes, (a) and (b), involve the interaction of electrons with a thermal bath of magnons on the right hand side of the junction and are responsible for transferring electrons in opposite directions. The process (a) begins with a $\uparrow$ electron on the left with wavevector $\mathbf{k}_{\mathrm{L}} = (\mathbf{k}_{\mathrm{L}}^{\|}, k_{\mathrm{L}}^{z})$ [with occupation number $n_{\mathrm{L}}(\mathbf{k}_{\mathrm{L}})$], which tunnels through the barrier into an intermediate virtual $\uparrow$ state on the right $(\mathbf{k}_{\mathrm{R}} = (\mathbf{k}_{\mathrm{L}}^{\|}, k_{\mathrm{R}}^{z}))$. Then, this electron flips spin by emitting a magnon with wavevector $\mathbf{q}$ [this process is stimulated by the occupancy factor of thermal magnon excitations $1 + N_{\mathrm{R}}(\mathbf{q})$], and, thus, incorporates itself into the majority spin band on the right, provided the final $\downarrow$ state $(\mathbf{k}' = \mathbf{k}_{\mathrm{R}} - \mathbf{q})$ is not occupied [which has probability $1 - n_{\mathrm{R}}(\mathbf{k}_{\mathrm{R}} - \mathbf{q})$]. The process (b) is the reverse to the process (a). It begins with a $\downarrow$ electron on the right with wavevector $\mathbf{k}' = \mathbf{k}_{\mathrm{R}} - \mathbf{q}$, that absorbs a magnon, flips its spin and moves into a virtual minority-spin state on the right. Then, it tunnels into an empty final state in the majority spin band in the left reservoir. The balance between these two processes contributes to the total current as

$$\begin{aligned} I_{\mathrm{ab}} = -4\pi^2 \frac{e}{h} \int_{-\infty}^{+\infty} d\epsilon \sum_{\mathbf{k}_{\mathrm{L}}\mathbf{k}_{\mathrm{R}}\mathbf{q}} |A_{\mathbf{k}_{\mathrm{L}},\mathbf{k}_{\mathrm{R}}}|^2 \\ \times \delta(\epsilon - \epsilon_{\mathbf{k}_{\mathrm{L}}})\, \delta(\epsilon - eV - \epsilon_{\mathbf{k}_{\mathrm{R}}-\mathbf{q}} - \omega_{\mathbf{q}}) \\ \times \{ n_{\mathrm{L}}(\mathbf{k}_{\mathrm{L}})\, [1 - n_{\mathrm{R}}(\mathbf{k}_{\mathrm{R}} - \mathbf{q})]\, [1 + N_{\mathrm{R}}(\mathbf{q})] - \\ - [1 - n_{\mathrm{L}}(\mathbf{k}_{\mathrm{L}})]\, n_{\mathrm{R}}(\mathbf{k}_{\mathrm{R}} - \mathbf{q}) N_{\mathrm{R}}(\mathbf{q}) \}\,, \end{aligned} \qquad (3)$$

where $n_{\mathrm{L/R}}(\mathbf{k}) = [\exp\{(\epsilon_{\mathbf{k}} - \epsilon_{\mathrm{F}}^{\mathrm{L/R}}\}/k_B T_{\mathrm{L/R}}) + 1]^{-1}$, $\epsilon_{\mathrm{F}}^{\mathrm{L}} - \epsilon_{\mathrm{F}}^{\mathrm{R}} = -eV$, $N_{\mathrm{L/R}}(\mathbf{q}) = [\exp(\omega_{\mathbf{q}}/k_B T_{\mathrm{L/R}}) - 1]^{-1}$, and $T_{\mathrm{L/R}} = T \pm \Delta T/2$.

Two other processes shown in Figure 1(c) and (d) involve emission/absorption of magnons on the left hand side of the junction. Their contribution to the total current is

$$\begin{aligned} I_{\mathrm{cd}} = -4\pi^2 \frac{e}{h} \int_{-\infty}^{+\infty} d\epsilon \sum_{\mathbf{k}_{\mathrm{L}}\mathbf{k}_{\mathrm{R}}\mathbf{q}} |A_{\mathbf{k}_{\mathrm{L}},\mathbf{k}_{\mathrm{R}}}|^2 \\ \times \delta(\epsilon - eV - \epsilon_{\mathbf{k}_{\mathrm{R}}})\, \delta(\epsilon - \epsilon_{\mathbf{k}_{\mathrm{L}}-\mathbf{q}} - \omega_{\mathbf{q}}) \\ \times \{ -n_{\mathrm{R}}(\mathbf{k}_{\mathrm{R}})\, [1 - n_{\mathrm{L}}(\mathbf{k}_{\mathrm{L}} - \mathbf{q})]\, [1 + N_{\mathrm{L}}(\mathbf{q})] + \\ + [1 - n_{\mathrm{R}}(\mathbf{k}_{\mathrm{R}})]\, n_{\mathrm{L}}(\mathbf{k}_{\mathrm{L}} - \mathbf{q}) N_{\mathrm{L}}(\mathbf{q}) \}\,. \end{aligned} \qquad (4)$$

After combining them together into an expression for the total current $I = I_{ab} + I_{cd}$, and, then, performing summation over wave numbers and integration over initial electron energies, we arrived at the following expression

$$I = +\frac{3}{4} \left( \frac{G_P}{\xi e} \right) \left( \frac{k_B T}{\omega_D} \right)^{3/2} [a\, eV - b\, k_B \Delta T]\,, \qquad (5)$$

where $a = 3\Gamma(3/2)\zeta(3/2)$, $b = (5/2)\Gamma(5/2)\zeta(5/2)$, $\Gamma(x)$ is the gamma function, and $\zeta(x)$ is Riemann's zeta function. Here, all properties of the interface are incorporated into a single parameter $G_P$ which coincides with the linear conductance of the same mesoscopic junction in the P configuration. For a flat, clean barrier of area $A$, where the parallel component of momentum is conserved upon tunneling, we consider the tunneling matrix element to have the form $t_{\mathbf{k},\mathbf{k}'} = \delta_{\mathbf{k}_{\|},\mathbf{k}'_{\|}}\, t\, |h^2 v_L^z v_R^z / L^2|^{1/2}$,

which gives $G_P \approx 4\pi^2(e^2/h)\,|t|^2\,(A\Pi_+/h^2)$ where $v^z_{L(R)}$ is the perpendicular component of velocity on the left (right) side, $L$ is the length of an electrode, $t$ is the barrier transparency, and $\Pi_+$ is the area of the maximal cross-section of the Fermi surface of majority electrons in the plane parallel to the interface. When deriving Eq. (5), we also assumed a quadratic magnon dispersion, $\omega_q = Dq^2$, and $k_BT \ll \omega_D$, where $\omega_D = D(6\pi^2/v)^{2/3}$ is the Debye magnon energy, and $v$ is the volume of a unit cell.

The thermopower coefficient $S = -V/\Delta T$ can be found by setting the total current in Eq. (5) to zero and determining the voltage created by the temperature difference. As a result, the tunneling conductance $G_P$ cancels from the final answer, and, in the antiparallel configuration, $S_{AP} \approx -0.64k_B/e$.[13] In contrast to the AP configuration, magnon-assisted tunneling cannot contribute to the electron transfer between two electrodes in the P configuration, since both initial and final electron states should have the same spin polarization in order to belong to the majority bands in both of the reservoirs. As a result, the linear conductance of such a junction is formed without the involvement of magnon-assisted processes, and the thermopower may only appear due to the energy-dependent electron tunneling density of states, having the order of magnitude of $S_P \sim (k_B/e)(k_BT/\epsilon_F)$.

A generalization to conventional ferromagnetic metals of the proposed theory of the magnon-assisted (ma) tunneling contribution to the thermopower yields

$$S^{\mathrm{ma}}_{AP} = -(k_B/e)\,g\,\theta; \quad S^{\mathrm{ma}}_P = 0, \tag{6}$$

$$g \approx \frac{1.7}{\xi}\left(\frac{k_BT}{\omega_D}\right)^{3/2};\ \theta = \begin{cases} (\Pi_+ - \Pi_-)/\Pi_-\,, & \text{flat} \\ (\Pi_+^2 - \Pi_-^2)/(\Pi_+\Pi_-), & \text{diff} \end{cases}$$

where $\Pi_\pm$ is the area of the maximal cross-section of the Fermi surface of majority/minority electrons in the plane parallel to the interface ($\Pi_+ > \Pi_-$), $\xi$ is the spin of localized moments, and $\omega_D$ is the magnon bandwidth. The function $g(T)$ is proportional to the fractional change in the net magnetization due to thermal magnons (Bloch's $T^{3/2}$ law) and the function $\theta$ is written for both a flat, clean interface ('flat') and a diffusive tunnel barrier ('diff'). This result was obtained after some amendments to the above analysis were made. First, the linear conductance in the AP configuration is not suppressed because an elastic tunneling channel is opened between the majority band on one side and the minority band on the other, which reduces the thermopower. Secondly, for the AP configuration, in addition to the magnon-assisted tunneling processes that enable transitions from majority initial to majority final states via an intermediate minority state (as described already for the half-metallic case and shown in Fig. 1), one should take into account the possibility of magnon-assisted tunneling processes that enable transitions from minority initial to minority final states via an intermediate majority state. A transition via a majority (minority) intermediate state results in the transfer of electrons in the same (opposite) direction as the net polarization transfer between two baths of magnons so that the additional processes partially compensate the thermally excited currents.

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[13] The sign of the thermopower coefficient is specified for the electron (charge $-e$) transfer process and under the assumption that the exchange between conduction band and core electrons has a ferromagnetic sign. For antiferromagnetic exchange, the sign of the thermopower would be opposite. One can verify this statement by taking into account that processes, described for example in Fig. 1(a) and (b), would involve magnons on the opposite side of the junction, hence the current $I_{\mathrm{ab}}$ would be determined by magnon occupation numbers $N_{\mathrm{L}}(\mathbf{q})$.